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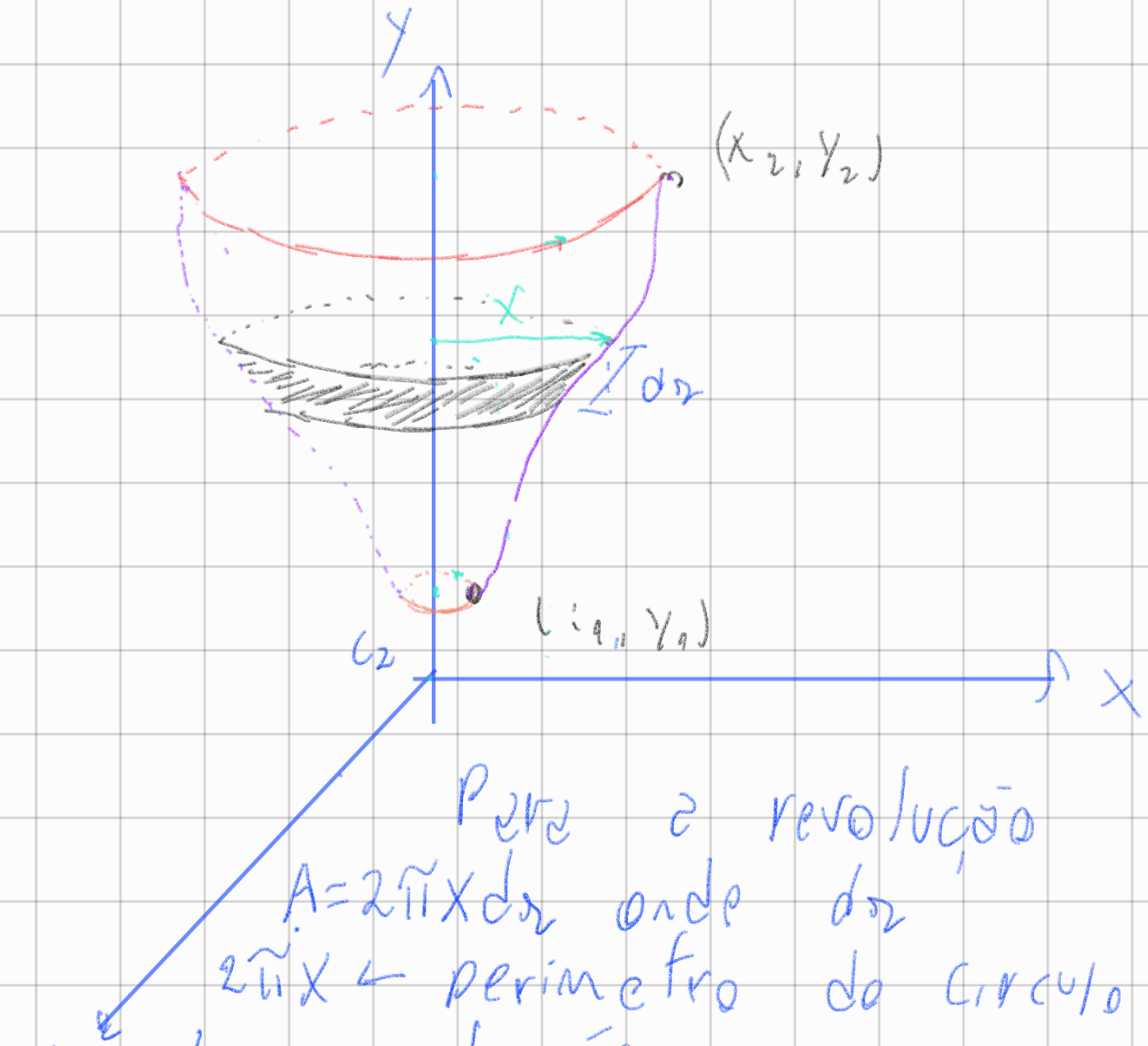
1- Consider the surface generated by revolving a line connecting two fixed points (x_1, y_1) and (x_2, y_2) about an axis coplanar with the two points. Find the equation of the line connecting the points such that the surface area generated by the revolution (i.e., the area of the surface of revolution) is a minimum.



Para a revolução

$A = 2\pi x\, ds$ onde ds

$2\pi x$ ← perímetro do círculo de revolução

ds é a "altura"

$$\frac{ds^2}{dx^2} = \frac{dx^2}{dx} + \frac{dy^2}{dx^2}$$

$$\dot{y} = \frac{dy}{dx}$$

$$\sqrt{ds^2} = \sqrt{(1+\dot{y}^2)}\sqrt{dx^2}$$

$$ds = \sqrt{1+\dot{y}^2}\,dx$$

$$A = 2\pi x\left(\sqrt{1+\dot{y}^2}\right)dx$$

Se deixa com dx pois o Raio x varia conforme ds desloca na linha que liga os pontos

$$A = 2\pi\int \underbrace{x\sqrt{1+\dot{y}^2}}\,dx$$

$$\Rightarrow f$$

função que devo encontrar o mínimo

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial \dot{y}}\right) = 0 \qquad \frac{\partial f}{\partial y} = 0$$

Fazendo $\frac{\partial f}{\partial \dot{y}} \times \sqrt{1+\dot{y}^2}$

$$\frac{\partial f}{\partial \dot{y}} \times \sqrt{1+\dot{y}^2} \rightarrow x(1+\dot{y}^2)^{1/2}$$

$$\frac{\partial f}{\partial \dot{y}} = \frac{x}{2}(1+\dot{y}^2)^{-1/2}(0+2\dot{y})$$

$$= \frac{1}{2} \cdot \frac{x \cdot 2\dot{y}}{\sqrt{1+\dot{y}^2}}$$

$$\frac{\partial f}{\partial \dot{y}} = \frac{x\dot{y}}{\sqrt{1+\dot{y}^2}}$$

$$\frac{d}{dx}\left(\frac{x\dot{y}}{\sqrt{1+\dot{y}^2}}\right) = 0$$

$$\frac{x\dot{y}}{\sqrt{1+\dot{y}^2}} = a$$

onde a é uma de integração muito menor que o valor mínimo de x

$$\frac{x\dot{y}}{\sqrt{1+\dot{y}^2}} = c \qquad \text{isolando } \dot{y}$$

$$(x\dot{y})^2 = \left(c\sqrt{1+\dot{y}^2}\right)^2$$

$$x^2\dot{y}^2 = c^2(1+\dot{y}^2)$$

$$x^2\dot{y}^2 = c^2 + c^2\dot{y}^2$$

$$c^2 = x^2\dot{y}^2 - c^2\dot{y}^2$$

$$c^2 = \dot{y}^2(x^2 - c^2)$$

$$\sqrt{\dot{y}^2} = \sqrt{\frac{c^2}{(x^2-c^2)}}$$

$$\dot{y} = \frac{c}{\sqrt{x^2-c^2}} \qquad \dot{y} = \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{c}{\sqrt{x^2-c^2}}$$

$$\frac{dy}{dx} = \frac{a}{\sqrt{x^2 - a^2}}$$

$$\int dy = \int \frac{a}{\sqrt{x^2 - a^2}}\, dx$$

$$y = a \int \frac{1}{\sqrt{x^2 - a^2}}\, dx$$

integral
tabel 2 da

$$y = a\left[\operatorname{arccosh}\left(\frac{x}{a}\right)\right] + c$$

$$x = a\cosh\left(\frac{y-b}{a}\right)$$

0